Application of Point Spread Function on Satellite Signature Effect

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Abstract

The point spread function (PSF) of incoherence imaging system is used into the study of satellite signature effect in this paper. The formula of the PSF for laser satellite is deduced. The PSF for Lageos and the laser return pulse intensity distribution is calculated. The calculation of CoM and the estimate of ranging precision (RMS) for Lageos are also given.

1. Point spread function (PSF) of incoherence imaging system

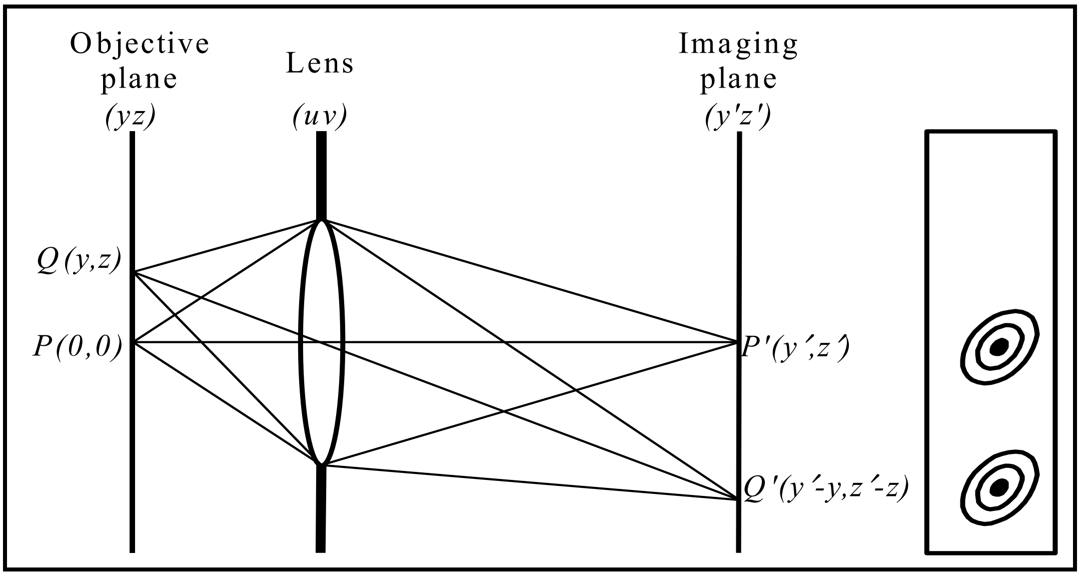

Figure 1: PSF caused by pupil diffraction

In Figure 1, let the PSF of point P is h (y′,z′), the PSF of point Q is h (y′-y,z′-z).

On the assumption that the intensity of the objective is O(y,z), the intensity of its image is

$$I(y',z') = \iint O(y,z)h(y'-y,z'-z)dydz \qquad (1)$$

This is just a convolution expression.

2. The formula of the PSF for laser satellite

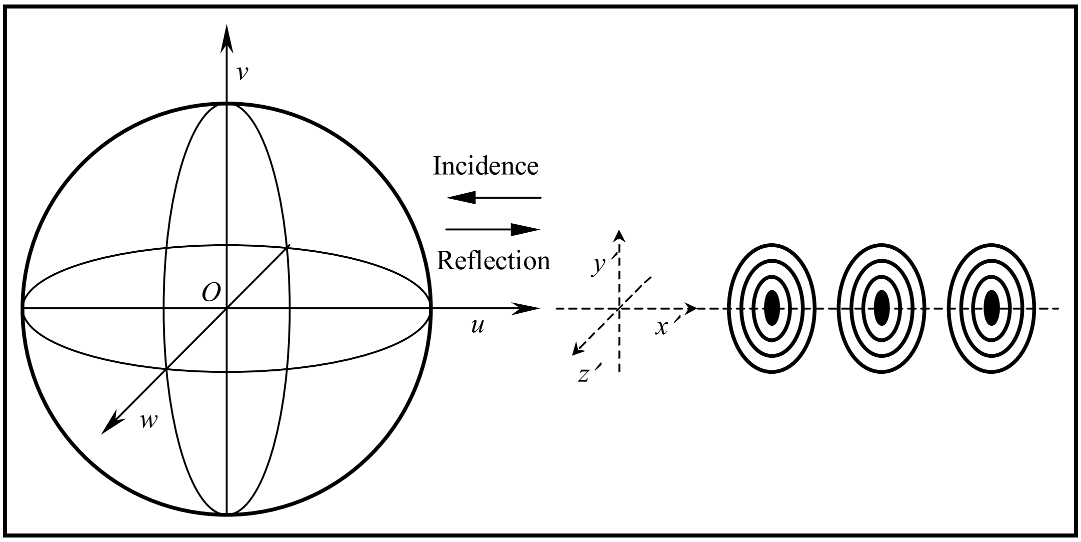

Figure 2: Coordinate frame of satellite diffraction system and isoplanatic patterns in diffraction fields

In Figure 2, there are three following cartesian coordinate frames:

xyz—the coordinate frame of the SLR system (objective plane)

uvw—the coordinate frame of the laser satellite (pupil)

x′y′z′—the coordinate frame of the diffraction field (imaging plane)

The intensity of pulse at far distance is the function of only x, i.e. O(x). Suppose the PSF of the corner-cube at origin (u=v=w=0) is h(x′,y′,z′), then the PSF of the no.i corner-cube at (u_i,v_i,w_i) is h(x′-u_i,y′-v_i,z′-w_i). Denoting the location of corner-cube with δ(u), which is the Dirac function. Hence the PSF for the whole pupil of satellite is as follow

$$H(x',y',z') = \sum_i \delta(x'-u_i)h_i(y'-v_i,z'-w_i) \qquad (2)$$

The intensity of the return signals from satellites in the far field is the convolutions of O(x) and H(x′,y′,z′):

$$I(x',y',z') = \iiint O(x'-x)\cdot\sum_i \delta(x'-u_i)h_i(y'-v_i,z'-w_i)dxdydz \qquad (3)$$

where the integration vs dydz is the sum of the area of all pupils (corner-cubes). Suppose A_0 is the area of the front corner-cube and the relative area of the no.i corner-cube is $\gamma(u_i)$, then

$$I(x',y',z') = A_0\int O(x'-x)\sum_i \delta(x-u_i)\gamma(u_i)h_i(y'-v_i,z'-w_i)dx \qquad (4)$$

As we know the intensity of diffraction is proportion to the area of the aperture and to the PSF, hence the PSF is also proportion to the latter.

So $$h_i(y'-v_i,z'-w_i)=\gamma(u_i)h_0(y',z') \tag{5}$$

and $$I(x',y',z')=A_0h_0(y',z')\int O(x'-x)\sum_i\delta(x-u_i)\gamma^2(u_i)dx \tag{6}$$

then $$H(x',y',z')=A_0h_0(y',z')\sum_i\delta(x'-u_i)\gamma^2(u_i) \tag{7}$$

After the integration of H(x′,y′,z′) vs dy′dz′, we can obtain the one dimension PSF

$$H(x')=A_0\sum_i\delta(x'-u_i)\gamma^2(u_i)\iint h_0(y',z')dy'dz' \tag{8}$$

then $$H(x')=KA_0^2\sum_i\delta(x'-u_i)\gamma^2(u_i) \tag{9}$$

Integration vs dx′

$$\int H(x')dx'=\int KA_0^2\sum_i\delta(x'-u_i)\gamma^2(u_i)dx'=KA_0^2\sum_i\gamma^2(u_i) \tag{10}$$

Unify the above formula and put into (9), we got

$$H(x')=\frac{\sum_i\delta(x'-u_i)\gamma^2(u_i)}{\sum_i\gamma^2(u_i)} \tag{11}$$

This is just the PSF for one dimension of laser satellite.

In order to obtain the relative area $\gamma(u_i)$, we can use the incidence angle ϕ_i for the no.i corner-cube. The relative area $\gamma(u_i)$ can be expressed with the incidence angle ϕ_i, i.e. $u=X(\phi)$, and the relative area is $\eta(\phi_i)$, then we can get $\gamma(u_i)$ from $\eta(\phi_i)$. The PSF in $\eta(\phi_i)$ is

$$H(x')=\frac{\sum_i\delta(x'-X(\phi_i))\eta^2(\phi_i)}{\sum_i\eta^2(\phi_i)} \tag{12}$$

Let $$h_U(u_i)=\gamma^2(u_i)\Big/\sum_i\gamma^2(u_i) \tag{13}$$

$$h_\Phi(\phi_i)=\eta^2(\phi_i)\Big/\sum_i\eta^2(\phi_i) \tag{14}$$

where $h_U(u_i)$ and $h_\Phi(\phi_i)$ can be called the point spread distribution core of the no.i corner-cube in variables u_i and ϕ_i respectively.

If the distribution of the corner-cubes can be looked as continuous, like Lageos. Let $D_U(u)$ and $D_\Phi(\phi)$ denoting the density of the corner-cubes, (12) can be transformed into the following expressions

$$H_U(u)=\frac{D_U(u)\gamma^2(u)}{\int_{[u]} D_U(u)\gamma^2(u)du} \tag{15}$$

$$H_\Phi(\phi)=\frac{D_\Phi(\phi)\eta^2(\phi)}{\int_{[\Phi]} D_\Phi(\phi)\eta^2(\phi)d\phi} \tag{16}$$

$$\begin{aligned} H(x') &= \int_{[U]} \delta(x'-u)H_U(u)du = H_U(x') \\ &= \int_{[\Phi]} \delta(x'-X(\phi))H_\Phi(\phi)d\phi = H_\Phi(\phi)\Big/\left\|\frac{dX(\phi)}{d\phi}\right\|_{\phi=X^{-1}(x')} \end{aligned} \tag{17}$$

where $X^{-1}(x')$ is the inverse function of $X(\phi)$.

3. Example: PSF of Lageos and its application

With the idea of PSF, we can research satellite signature effects. For example, the PSF of Lageos can be used to construct the Center-of-Mass (CoM) model for calculating its CoM values, which was determined by the method of probability by other authors.

Another advantage of PSF is that the calculation of the pulse intensity reflected by satellite becomes quite simple. The intensity distribution is the convolution of the initial pulse intensity and the PSF of satellite.

Construct the coordinate frame of Lageos as follows (see Figure 3).

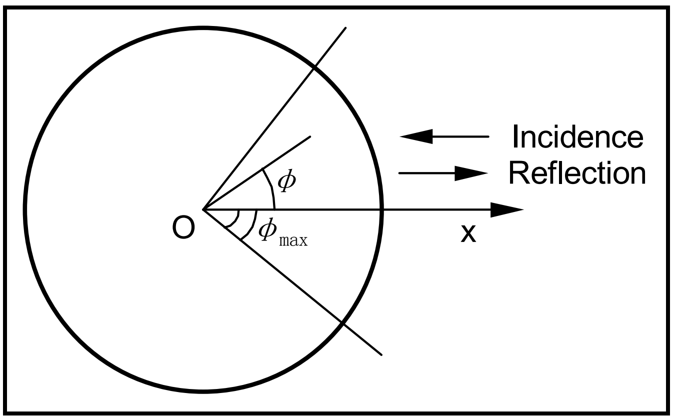

Figure 3: Sketch for coordinate frame of the PSF model of Lageos

The function $X(\phi)$ is

$$X(\phi)=R_s\cos(\phi)-L\sqrt{n^2-\sin^2(\phi)} \tag{18}$$

where R_s is the distance from the center of satellite to the front face of corner-cube, L is the height of corner-cube, n is the refractive index.

For a corner-cube, the relative area vs incidence angle ϕ is

$$\eta(\phi)=\begin{cases}1-\phi/\phi_{\max} & (\phi\le\phi_{\max})\\ 0 & (\phi>\phi_{\max})\end{cases} \tag{19}$$

where ϕ_{max} is the cut-off angle. The density of cubes is

$$D_\Phi(\phi)=N\sin(\phi)/2 \tag{20}$$

where N is the total number of cubes on the surface of Lageos. Then the point spread core function vs ϕ is

$$H_\Phi(\phi)=\frac{\eta^2(\phi)\cdot\sin(\phi)}{\int_0^{\pi/2}\eta^2(\phi)\cdot\sin(\phi)\cdot d\phi}=\frac{\eta^2(\phi)\cdot\sin(\phi)}{\int_0^{\phi_{\max}}\eta^2(\phi)\cdot\sin(\phi)\cdot d\phi} \tag{21}$$

transform to the core function with the variable of x is

$$H_X(x)=H_\Phi(\phi)\Big/\left|\frac{d\mathrm{X}(\phi)}{d\phi}\right|\Bigg|_{\phi=X^{-1}(x)}=\frac{H_\Phi(\phi)}{\sin(\phi)\cdot\left(R_s-\frac{L\cos(\phi)}{\sqrt{n^2-\sin^2(\phi)}}\right)}\Bigg|_{\phi=X^{-1}(x)} \tag{22}$$

Suppose the initial pulse intensity is

$$O(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{23}$$

where σ is the standard error of the distribution. For a pulse of 30ps width, σ=3.822mm.

Then the pulse intensity reflected by Lageos, with the formula (1), is

$$I(x)=O(x)*H(x)=\int_{[X(\phi)]}O(x-X(\phi))H_X(X(\phi))dX(\phi)$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\int_0^{\phi_{\max}}\exp\left(-\frac{(x-X(\phi))^2}{2\sigma^2}\right)H_\Phi(\phi)d\phi \tag{24}$$

Take the parameters of Lageos, R_s—298mm, L—27.84mm, n—1.455 and ϕ_{max}—0.75rad(43°), the final result of I(x) is shown in Figure 4.

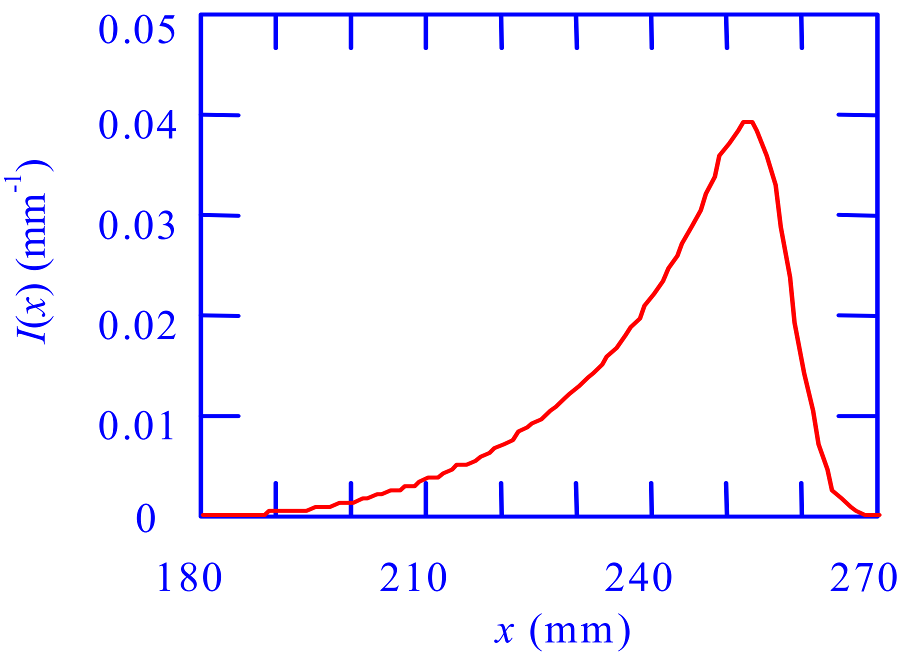

Figure 4: Pulse intensity distribution reflected by Lageos

4. Calculation of CoM and RMS for Lageos

We consider the following factors besides the Lageos signature effect:

1) Poisson distribution for photoelectrons produced from photosensitive area of detector.
2) Time jitter of detector can be denoted as a normal distribution, assuming 20 ps for the C-SPAD detector.

Then we can describe the transformation of laser pulse waveform during the Lageos ranging process as Figure 5. In Figure 5, (a) shows the output pulse from laser, (c) shows the reflection pulse from Lageos, (d) shows the pulse waveforms from the photosensitive area of detector, and (f) shows the pulse waveforms from the output of detector. Q in (d) and (f) of Figure 5 is average number of the received photoelectrons.

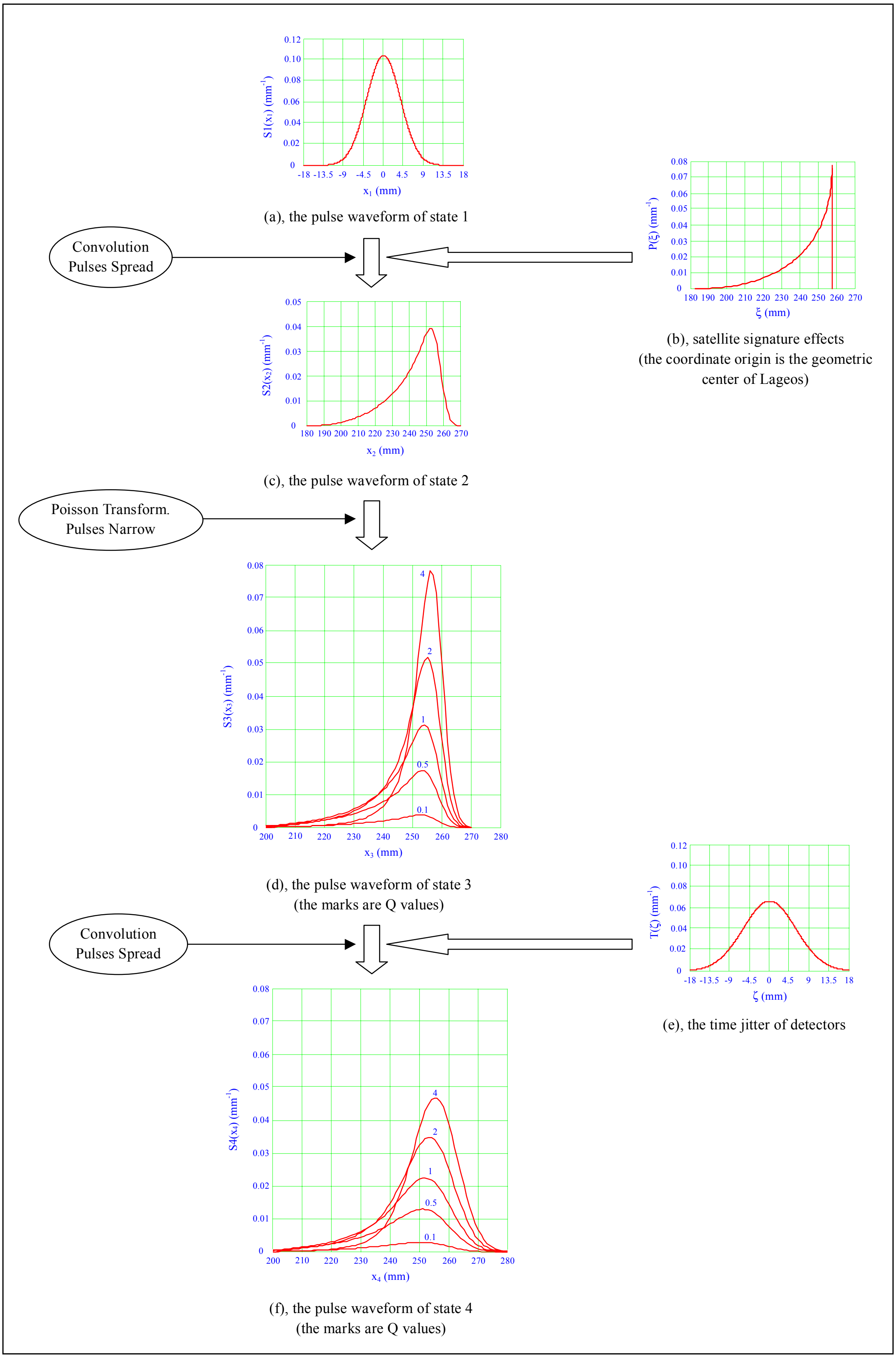

Figure 5: Calculation results of pulse waveform during Lageos ranging

Based on the above calculation, we can estimate the RMS from the standard variation of the random variables. Furthermore, as long as taking account of time jitter of the time interval counter, the total precision of the SLR system can be worked out. At Shanghai Observatory, the time jitter of HP5370 counter is 35 ps. The calculation results of CoM and RMS values for single shot precision are listed in Table 1 and Figure 6. In Table 1, RMS (x_4) is come from the standard variation of pulse waveforms from the output of detector, RMS (Total) is for the whole ranging system, including time jitter of counter. Therefore, we have the following conclusions:

1) For Lageos, the different returned pulse strengths will cause serious variation of CoM value. When the average photoelectrons vary from 0.1 to 20, the shift of CoM would reach 17 mm. For single photon ranging system, the CoM value for Lageos seems to be 243-244 mm. While 252-253 mm for multi-pe system seems suitable.
2) For Lageos, if the average returned signals are below 2 pe, the single shot RMS can not be better than 8 mm. Even for the picosecond timer system, the single shot RMS still will be around 6 mm.

Q (pe)	CoM (mm)	RMS(x_4)(mm)	RMS(total)(mm)
0.1	242.64	7.54	9.19
0.5	244.12	7.32	9.00
1	245.88	6.99	8.74
2	248.96	6.27	8.17
4	253.25	4.94	7.21
10	257.89	3.57	6.35
20	259.99	3.32	6.21

Table 1: CoM and RMS values of Lageos in consideration of returned signal strength and signal detection

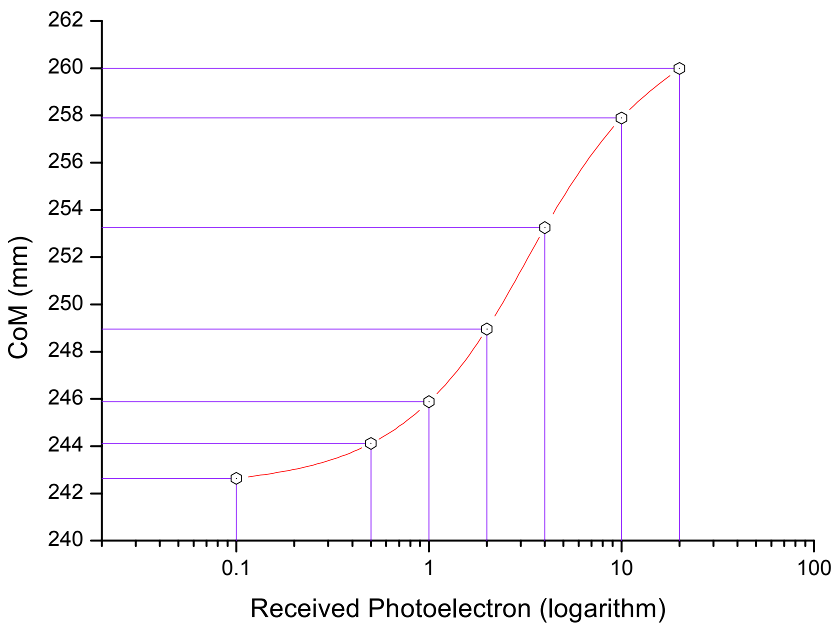

Figure 6: CoM values versus returned signal strength from Lageos

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